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Statistical mechanics of competitive exclusion of ecological communities

While the competitive exclusion principle or Gauze' s law has been regarded as one of the most fundamental laws of ecology, it has also aroused controversy since it is rarely observed in real ecosystems and there are many exceptions such as coexistence of large numbers of plankton species within small regions of open sea. Although many types of explanations for such exceptions has been proposed, e.g., life history, body size, spatial heterogeneity, trophic interactions, multiple resource competition, competition-colonization trade-offs, etc., there are few mathematical models incorporating such factors. Here we consider the replicator equations

$$\frac{dx_i}{dt} = x_i \left(f_i - \bar{f} \right), \ i \in [1, \cdots, 2N],$$

with $2N \times 2N$ random symmetric interaction matrix $\mathbf{A}$ which has four blocks.

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}^1 & \mathbf{A}^{1\to2} \\ \mathbf{A}^{2\to1} & \mathbf{A}^2 \end{pmatrix}$$

where each block matrix $\mathbf{A}^1$, $\mathbf{A}^{1\to2}$, $\mathbf{A}^{2\to1}$ and $\mathbf{A}^2$ is a $N \times N$ symmetric random matrix correlating each other:

$$Corr(\mathbf{A}^1_{ij}, \mathbf{A}^2_{ij}) = \frac{W^2}{W^2+V^2},$$

$$Corr(\mathbf{A}^1_{ij}, \mathbf{A}^{1\to2}_{ij}) = Corr(\mathbf{A}^2_{ij}, \mathbf{A}^{1\to2}_{ij}) = \frac{W\bar{W}}{\sqrt{W^2+V^2}\sqrt{\bar{W}^2+\bar{V}^2}},$$

$$Corr(\mathbf{A}^1_{ij}, \mathbf{A}^{2\to1}_{ij}) = Corr(\mathbf{A}^2_{ij}, \mathbf{A}^{2\to1}_{ij}) = \frac{W\bar{W}}{\sqrt{W^2+V^2}\sqrt{\bar{W}^2+\bar{V}^2}},$$

$$Corr(\mathbf{A}^{1\to2}_{ij}, \mathbf{A}^{2\to1}_{ij}) = \frac{\bar{W}^2}{\bar{W}^2+\bar{V}^2}.$$

The control parameters W, $\bar{W}$, V and $\bar{V}$ are introduced for the correlation among the matrices as:

$$\mathbf{A}^1_{ij} = \mathbf{A}^1_{ji} = Wa^0_{ij} + Va^1_{ij},\ \mathbf{A}^2_{ij} = \mathbf{A}^2_{ji} = Wa^0_{ij} + Va^2_{ij},$$
$$\mathbf{A}^{1\to2}_{ij} = \mathbf{A}^{1\to2}_{ji} = \mathbf{A}^{2\to1}_{ij} = \mathbf{A}^{2\to1}_{ji} = \bar{W}a^0_{ij} + \bar{V}a^{1\to2}_{ij},$$

where a^0_{ij}, a^1_{ij}, a^2_{ij} and $a^{1\to2}_{ij}$ is generated by i.i.d. standard normal distribution. The block structure of the interaction matrix $\mathbf{A}$ incorporates, e.g., spatial heterogeneity (two patches), two body sizes (two levels of age structure) in the replicator equations. The main result of the statistical mechanics analysis is that the larger the correlation, that is, more species compete for a same resource, the lower the diversity, which means community-level competitive exclusion in the large-scale ecosystem model. The community-level competitive exclusion has been proved for two elementary hypercycles [1] but the present model may be the first example for more complex community. We will present a detail of the model and the relation between the diversity and the correlation.

References

[1] J. Hofbauer, Competitive Exclusion of Disjoint Hypercycles, *Zeitschrift für Physikalische Chemie*, 216, 35-39