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Superselection rule from measurement theory

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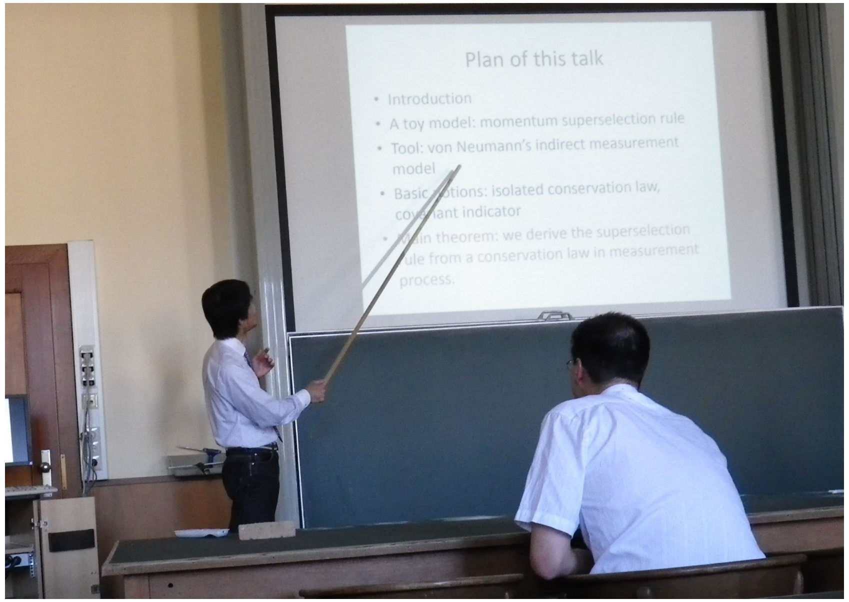
Introduction of myself

- 1967, born in Nagoya, that is a city located between Tokyo and Osaka. Toyota city is in the same prefecture.
- 1990, finish the undergraduate course of Department of Applied Physics, Nagoya University (adviser: Haga)
- 1995, got PhD. degree, at E-lab, Department of Physics, Nagoya University (adviser: Ohnuki, Kitakado, Sanda, Yamawaki)
- JSPS Posdoc at University of Tokyo (host: Eguchi)
- Got job at Kyoto University, Osaka City University, again Kyoto University
- 2011, came back to Nagoya University

Visits to Germany

- Fist visit to Max-Planck Institute in Heidelberg
- Second visit to University of Göttingen to deliver seminars at Dr. Karl-Henning Rehren's and Dr. Ralf Meyer's groups, in 2012.



Research interests

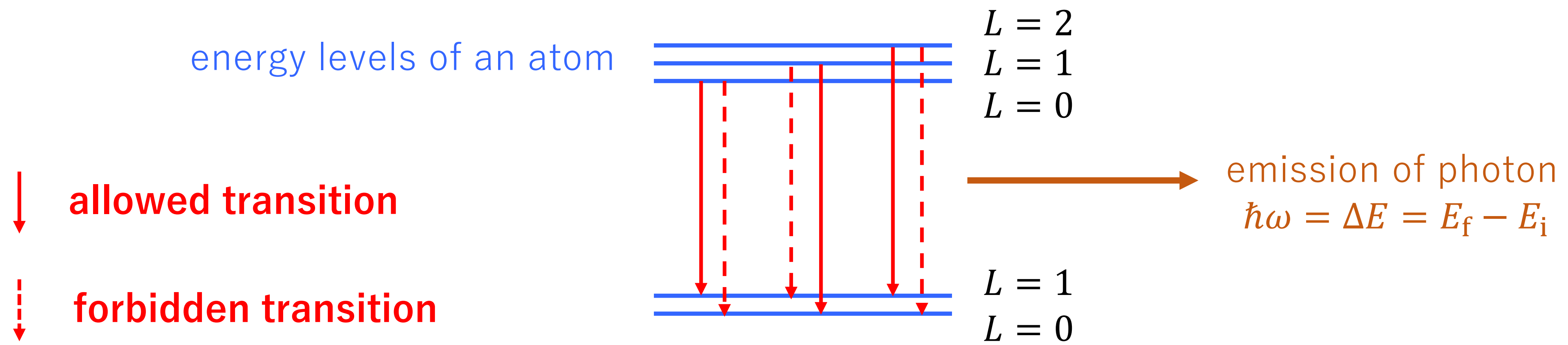
- Quantum Foundations
- Quantum Information Theory
- Dynamical Systems
- Statistical Mechanics
- Differential Geometry
- Category Theory
- Artificial Intelligence

Today's Topic

- Superselection rule in quantum theory from the viewpoint of measurement theory

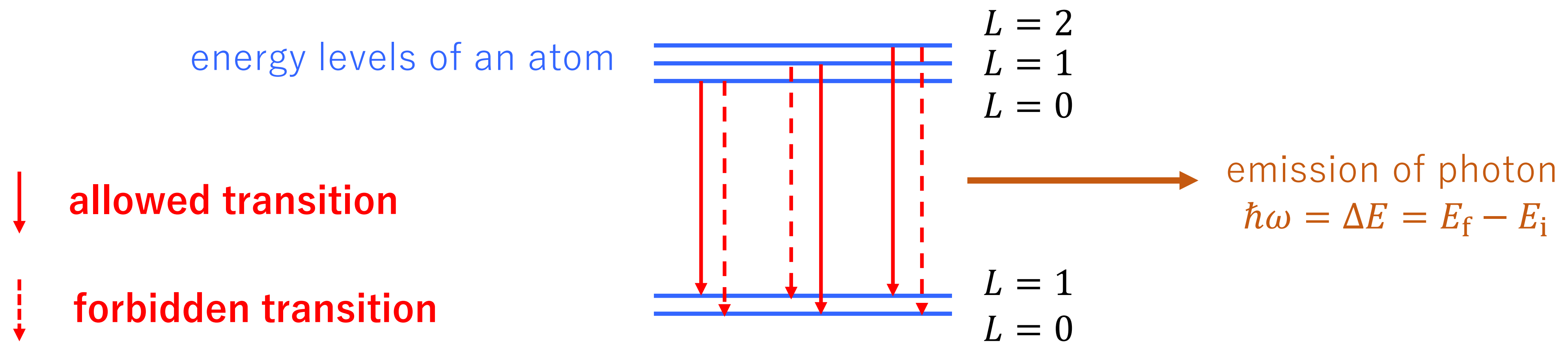
Selection rule

- Originally, the selection rule was found in spectrum of atoms.
- Some of photonic transitions do not occur even though they are allowed by energy conservation.
- **Selection rule**: $\Delta L = L_\mathrm{f} - L_\mathrm{i} = \pm 1$
- Some of transitions (for example $\Delta L = \pm 2$) are not allowed by angular-momentum conservation.



Selection rule

- **Some of transitions are selected to occur**: $\Delta L = L_\mathrm{f} - L_\mathrm{i} = \pm 1$
- In this case, the selection rule is another name of conservation law.
- A photon carries angular momentum by an amount of spin 1, and the total angular momentum should be conserved, and hence we observe the rule $\Delta L = \pm 1$.



What is the superselection rule?

- There are various appearance (even various formulations) of the superselection rule.
- Originally, it was found in the context of relativistic quantum field theory by Wick, Wigner, and Wightman (WWW) in 1952.

The statement of superselection rule

- **There is a superselection charge $\hat{Q}$.**
- **If a self-adjoint operator $\hat{A}$ represents a *truly observable* quantity, it must satisfy $[\hat{A}, \hat{Q}] := \hat{A}\hat{Q} - \hat{Q}\hat{A} = 0$.**
- By contraposition, **if $[\hat{A}, \hat{Q}] \neq 0$ for a self-adjoint operator $\hat{A}$,** ***$\hat{A}$ does not represent a truly observable quantity.***

- The **selection rule** selects observable transitions of an atom.
- The **superselection rule** selects observable operators of a system.

Example of application of the superselection rule

- complex scalar field: $\hat{\phi}(x)$
- conserved current: $\hat{j}_\mu(x) = -i\hat{\phi}^*\partial_\mu\hat{\phi} + i\left(\partial_\mu\hat{\phi}^*\right)\hat{\phi}, \quad \partial_\mu\hat{j}^\mu = 0$
- superselection charge: $\hat{Q} \coloneqq \int \hat{j}^0 d^3x$
- $\left[\hat{Q}, \hat{\phi}\right] = -i\hat{\phi} \neq 0 \implies \hat{\phi}$ itself is not observable.
- Neutral quantities like $\hat{\phi}^*\hat{\phi}$ and $\hat{j}_\mu$ can be observable.

History of the superselection rule

- Around 1950, physicists discussed definition of the parity transformation of the Dirac field $\psi(x)$.
- parity transformation: $\psi(\boldsymbol{x}, t) \mapsto \Pi\psi(\boldsymbol{x}, t) = e^{i\theta}\gamma^0\psi(-\boldsymbol{x}, t)$
- The phase factor $e^{i\theta}$ is not uniquely determined, it has ambiguity.
- Yang, Tiomno, Zharkov proposed four types $e^{i\theta} = \pm 1, \pm i$.
- 1951, Fermi discussed how to distinguish the intrinsic parity exprimentarily.
- Wick, Wigner and Wightman noticed that **the Dirac spinor itself cannot be measured, and hence we cannot determine its parity phase factor.**

Univalence superselection rule

- $\hat{Q} = \hat{R}(2\pi)$: rotation by 360 degree around arbitrary axis.
- A measurable quantity $\hat{A}$ must satisfy

$$\hat{R}(2\pi)^{\dagger}\hat{A}\,\hat{R}(2\pi) = \hat{A}$$

or equivalently, $[\hat{A}, \hat{Q}] = 0$.

- On the other hand, the Dirac spinor field $\hat{\psi}$ satisfies

$$\hat{R}(2\pi)^{\dagger}\,\hat{\psi}\,\hat{R}(2\pi) = -\hat{\psi}.$$

- Thus, the Dirac spinor field $\hat{\psi}$ itself is not a measurable quantity.
- However, $\hat{\psi}^{\dagger}\hat{\psi}$ is measurable.

Self-adjointness implies measurability?

- **For the Dirac field $\hat{\psi}$,**

$$\hat{\boldsymbol{\psi}} + \hat{\boldsymbol{\psi}}^\dagger, \qquad \boldsymbol{i}(\hat{\boldsymbol{\psi}} - \hat{\boldsymbol{\psi}}^\dagger)$$

  are formally self-adjoint, but it seems that they are non-measurable.
- The gauge invariant quantities

$$\hat{\psi}^\dagger \hat{\psi}, \qquad \hat{\psi}^\dagger \gamma^\mu \hat{\psi}$$

  are formally self-adjoint and measurable.
- **Not all of the self-adjoint operators represent observables?**

von Neumann's postulate (1932)

- After showing that every observable is representable by a self-adjoint operator, von Neumann argued that **it is appropriate to assume that there is a physical observable corresponding to each self-adjoint operator.**

  observable $\hat{A}$ $\Rightarrow$ (⇐?) **self-adjoint** $\hat{A}^{\dagger} = \hat{A}$

- Original German expression by von Neumann (1932)
  《... **es ist zweckmässig anzunehmen,** ...》
- English translation by Beyer (1955)
  《... **it is convenient to assume,** ...》
- English translation by Wightman (1995)
  《... **it is appropriate to assume,** ...》

Implication of the superselection rule

- **There exist some operators that are self-adjoint but do not correspond to measurable quantities.**
- For the complex scalar field $\hat{\phi}$,

$$\hat{\phi} + \hat{\phi}^\dagger, \qquad i(\hat{\phi} - \hat{\phi}^\dagger), \qquad \hat{\phi}\hat{\phi}\hat{\phi} + \hat{\phi}^\dagger\hat{\phi}^\dagger\hat{\phi}^\dagger$$

  are self-adjoint but non-measurable.
- For the Dirac field $\hat{\psi}$,

$$\hat{\psi} + \hat{\psi}^\dagger, \qquad i(\hat{\psi} - \hat{\psi}^\dagger)$$

  are formally self-adjoint but non-measurable.
- **In what sense they are non-measurable?**

Mathematical formulation of measurement

- Hilbert space of a **microscopic object system**: $\mathfrak{H} \ni |\alpha\rangle$
- Hilbert space of a **measuring apparatus**: $\mathfrak{K} \ni |\beta\rangle$
- Hilbert space of **the composite system**: $\mathfrak{H}\otimes\mathfrak{K} \ni |\alpha\rangle\otimes|\beta\rangle$
- **An observable of the object system** $\hat{A}$, self-adjoint operator on $\mathfrak{H}$
- **A meter observable of the apparatus** $\hat{M}$, self-adjoint operator on $\mathfrak{K}$
- A unitary operator $\hat{U}$ acting on $\mathfrak{H}\otimes\mathfrak{K}$ describes interaction of the two system and their time-evolution

$$|\alpha\rangle\otimes|\beta\rangle \;\mapsto\; \hat{U}|\alpha\rangle\otimes|\beta\rangle$$

- Read out a value of the meter $\hat{1}\otimes\hat{M}$ on the state $\hat{U}|\alpha\rangle\otimes|\beta\rangle$ for inferring the value of $\hat{A}$ on the state $|\alpha\rangle$.

Indirect-measurement model

- Read out a value of the meter $\hat{1}\otimes\hat{M}$ on the state $\hat{U}|\alpha\rangle\otimes|\beta\rangle$ for inferring the value of $\hat{A}$ on the state $|\alpha\rangle$.

$$\begin{array}{ccc} \mathfrak{H} & \longrightarrow & \mathfrak{H}\otimes\mathfrak{R}, \\ \hat{A}\downarrow & & \hat{1}\otimes\hat{M}\downarrow \\ \mathfrak{H} & \dashrightarrow & \mathfrak{H}\otimes\mathfrak{R} \end{array}$$

$$|\alpha\rangle \mapsto \hat{U}|\alpha\rangle\otimes|\beta\rangle$$

$\hat{A} = \int \lambda\, \hat{E}_A(d\lambda)$, $\hat{M} = \int \lambda\, \hat{E}_M(d\lambda)$ spectral decompositions

- POVM (probability-operator valued measure)

$$\hat{P}_M(d\lambda) := \left\langle \beta \middle| \hat{U}^\dagger \left(\hat{1}\otimes\hat{E}_M(d\lambda) \right) \hat{U} \middle| \beta \right\rangle$$

- In the ideal measurement, $\hat{E}_A(d\lambda) = \hat{P}_M(d\lambda)$. But we do not demand that the measurement is ideal.

von Neumann's model of measurement

- Hilbert space of a **microscopic object system**: $\mathfrak{H} = L^2(\mathbb{R})$
- Hilbert space of a **measuring apparatus**: $\mathfrak{K} = L^2(\mathbb{R})$
- Hilbert space of **the composite system**: $\mathfrak{H} \otimes \mathfrak{K}$
- **An observable of the object system** $\hat{x}$, position operator on $\mathfrak{H}$
- **A meter observable of the apparatus** $\hat{X}$, position operator on $\mathfrak{K}$
- A unitary operator $\hat{U} = \exp\left(-i\hat{x} \otimes \hat{P}/\hbar\right)$ with $\left[\hat{X}, \hat{P}\right] = i\hbar\hat{1}$ yeilds

$$\hat{U}^{\dagger}\left(\hat{1} \otimes \hat{X}\right)\hat{U} = \hat{X} + \hat{x}.$$

The meter $\hat{X}$ shifts by the amount of $\hat{x}$. Hence, we can read out the position of the object particle from the meter position.

- This is not the ideal measurement.

Covariance condition

- Most of measurements are not ideal.
- What condition should be satisfied by a “good” measurement?
- In a good measurement, the meter follows a change of the object.

Group action on the observables

- Assume that a group G acts on the object Hilbert space $\mathfrak{H}$ by a projective unitary representation $\hat{S}_g$ $(g \in G)$.
- The group G acts on the apparatus Hilbert space $\mathfrak{K}$ by a projective unitary representation $\hat{T}_g$ $(g \in G)$.
- Assume that there are sets of observables which transform

$$\hat{S}_g^\dagger \hat{A}_\mu \hat{S}_g = \sum_\nu D_{\mu\nu}^A(g)\hat{A}_\nu , \qquad \hat{T}_g^\dagger \widehat{M}_j \hat{T}_g = \sum_k D_{jk}^M(g)\widehat{M}_k$$

- For example, the group $\mathbb{R}$ acts on the position operator as

$$\hat{S}_a{}^\dagger \hat{x}\, \hat{S}_a = \hat{x} + a\hat{1}$$

Covariant measurement

- Require that, for arbitrary action of $g \in G$,

$$
\begin{array}{ccc}
\mathfrak{H} & \longrightarrow & \mathfrak{H}\otimes\mathfrak{R}, \qquad |\alpha\rangle \mapsto \hat{U}|\alpha\rangle\otimes|\beta\rangle \\
\hat{S}_g \downarrow & & \hat{S}_g\otimes\hat{T}_g \downarrow \\
\mathfrak{H} & \dashrightarrow & \mathfrak{H}\otimes\mathfrak{R}
\end{array}
$$

  the meter transforms covariantly

$$
\begin{aligned}
\hat{S}_g^\dagger\hat{U}^\dagger(\hat{1}\otimes\hat{M}_j)\hat{U}\hat{S}_g &= \hat{U}^\dagger(\hat{S}_g^\dagger\otimes\hat{T}_g^\dagger)(\hat{1}\otimes\hat{M}_j)(\hat{S}_g\otimes\hat{T}_g)\hat{U} \\
&= \hat{U}^\dagger(\hat{S}_g^\dagger\hat{1}\hat{S}_g\otimes\hat{T}_g^\dagger\hat{M}_j\hat{T}_g)\hat{U} \\
&= \hat{U}^\dagger(\hat{1}\otimes\hat{T}_g^\dagger\hat{M}_j\hat{T}_g)\hat{U}
\end{aligned}
$$

- Require also that the representations D^A and D^M are equivalent.

Main assumption: isolated conservation

Assume that the measurement process conserves the charges $\hat{Q}_\kappa$ which generate the unitary transformation $\hat{S}_\theta = e^{i\hat{Q}_\kappa\theta_\kappa}$ of the object system, namely, assume

$$\hat{U}\hat{S}_g = \hat{S}_g\hat{U}.$$

Then,

$$\hat{S}_g^\dagger\hat{U}^\dagger(\hat{1}\otimes\hat{M}_j)\hat{U}\hat{S}_g = \hat{U}^\dagger(\hat{1}\otimes\hat{T}_g^\dagger\hat{M}_j\hat{T}_g)\hat{U}$$

$$\hat{U}^\dagger\hat{S}_g^\dagger(\hat{1}\otimes\hat{M}_j)\hat{S}_g\hat{U} =$$

$$\hat{U}^\dagger(\hat{S}_g^\dagger\hat{S}_g\otimes\hat{M}_j)\hat{U} =$$

$$\hat{U}^\dagger(\hat{1}\otimes\hat{M}_j)\hat{U} =$$

Therefore, **the covariant meter must obey the trivial representation**.

Main result: superselection rule

The observable $\hat{A}$ of the object system that is measurable with a meter that is covariant under the group G generated by the charges $\hat{Q}_\kappa$ that is a conserved during the measurement process, must obey the trivial representation of the group:

$$\hat{S}_g^\dagger \hat{A} \hat{S}_g = \hat{A}$$

or

$$\left[\hat{Q}_\kappa, \hat{A}\right] = 0$$

Relation with the uncertainty relation

- If two self-adjoint operotors $\hat{A}$ and $\hat{B}$ do not commute, namely, if $[\hat{A}, \hat{B}] = 0$, there is no CONS (complete orthonormal system of the Hilbert space) that diagonalizes operotors $\hat{A}$ and $\hat{B}$ simultaneously.
- It is often said **"measurement of $\widehat{\boldsymbol{A}}$ inevitably disturbs the value of $\widehat{\boldsymbol{B}}$."**
- **If $\widehat{\boldsymbol{B}}$ is a conserved quantity, the value of $\widehat{\boldsymbol{B}}$ cannot be changed. In this case, is measurement of $\widehat{\boldsymbol{A}}$ imposible?**
- **Qualitative answer (Wigner, Araki, Yanase):**
  precise measurement of $\widehat{\boldsymbol{A}}$ is impossible. (This is a gentle version of the superselection rule.)
- **Quantitative answer (Ozawa):**

Definitions

- $\hat{A}$ and $\hat{B}$: observables self-adjoint operators on $\mathfrak{H} \ni |\alpha\rangle$
- $\widehat{M}$: meter, self-adjoint operator on $\mathfrak{R} \ni |\beta\rangle$
- $\widehat{U}$: unitary operator on $\mathfrak{H} \otimes \mathfrak{R} \ni |\alpha\rangle \otimes |\beta\rangle$
- expectation value $\langle \hat{A} \rangle \coloneqq \langle \alpha | \hat{A} | \alpha \rangle$
- standard deviation $\sigma(\hat{A}) \coloneqq \sqrt{\left\langle \left(\hat{A} - \langle \hat{A} \rangle \right)^2 \right\rangle} = \sqrt{\langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2}$

Ozawa's formulation of the uncertainty relation

- error operator $\hat{E} := \hat{U}^{\dagger}\hat{M}\hat{U} - \hat{A}$
- error in the measurement of $\hat{A}$, $\varepsilon(\hat{A}) := \sqrt{\langle \hat{E}^2 \rangle}$
- disturbance operator $\hat{D} := \hat{U}^{\dagger}\hat{B}\hat{U} - \hat{B}$
- disturbance associated with the measurement, $\eta(\hat{B}) := \sqrt{\langle \hat{D}^2 \rangle}$
- Ozawa's inequality

$$\varepsilon(\hat{A})\eta(\hat{B}) + \varepsilon(\hat{A})\sigma(\hat{B}) + \sigma(\hat{A})\eta(\hat{B}) \geq \frac{1}{2}\left|\langle[\hat{A}, \hat{B}]\rangle\right|$$

Proof of Ozawa's inequality (1/2)

By definitions,

- $\hat{E} \coloneqq \hat{U}^\dagger \hat{M} \hat{U} - \hat{A}$, $\hat{U}^\dagger \hat{M} \hat{U} = \hat{E} + \hat{A}$
- $\hat{D} = \hat{U}^\dagger \hat{B} \hat{U} - \hat{B}$, $\hat{U}^\dagger \hat{B} \hat{U} = \hat{D} + \hat{B}$
- $\left[\hat{M}, \hat{B}\right] = \left[\hat{1} \otimes \hat{M}, \hat{B} \otimes \hat{1}\right] = 0,$

Therefore

$$
\begin{aligned}
0 = \hat{U}^\dagger\left[\hat{M}, \hat{B}\right]\hat{U} &= \left[\hat{U}^\dagger \hat{M} \hat{U}, \hat{U}^\dagger \hat{B} \hat{U}\right] \\
&= \left[\hat{E} + \hat{A}, \hat{D} + \hat{B}\right] \\
&= \left[\hat{E}, \hat{D}\right] + \left[\hat{E}, \hat{B}\right] + \left[\hat{A}, \hat{D}\right] + \left[\hat{A}, \hat{B}\right]
\end{aligned}
$$

$$
\therefore \left[\hat{E}, \hat{D}\right] + \left[\hat{E}, \hat{B}\right] + \left[\hat{A}, \hat{D}\right] = -\left[\hat{A}, \hat{B}\right]
$$

Proof of Ozawa's inequality (2/2)

- The Kennard-Robertson inequality: $\sigma(\hat{A})\sigma(\hat{B}) \geq \frac{1}{2}|\langle[\hat{A},\hat{B}]\rangle|$
- $\varepsilon(\hat{A}) \coloneqq \sqrt{\langle\hat{E}^2\rangle} \geq \sqrt{\langle\hat{E}^2\rangle - \langle\hat{E}\rangle^2} = \sigma(\hat{E})$
- $\eta(\hat{B}) \coloneqq \sqrt{\langle\hat{D}^2\rangle} \geq \sqrt{\langle\hat{D}^2\rangle - \langle\hat{D}\rangle^2} = \sigma(\hat{D})$

From $[\hat{E},\hat{D}] + [\hat{E},\hat{B}] + [\hat{A},\hat{D}] = -[\hat{A},\hat{B}]$,

$$\sigma(\hat{E})\sigma(\hat{D}) + \sigma(\hat{E})\sigma(\hat{B}) + \sigma(\hat{A})\sigma(\hat{D}) \geq \sigma(\hat{A})\sigma(\hat{B})$$

$$\therefore\ \varepsilon(\hat{A})\eta(\hat{B}) + \varepsilon(\hat{A})\sigma(\hat{B}) + \sigma(\hat{A})\eta(\hat{B}) \geq \cdots \geq \frac{1}{2}|\langle[\hat{A},\hat{B}]\rangle|$$

Uncertainty relation with a conserved quantity

- If $\hat{B} = \hat{Q}_{\text{object}} + \hat{Q}_{\text{apparatus}} = \hat{Q}_{\text{object}} \otimes \hat{1} + \hat{1} \otimes \hat{Q}_{\text{apparatus}}$ is conserved, then the disturbance $\eta(\hat{B}) = 0$, and hence

$$\varepsilon(\hat{A})\sigma(\hat{Q}_{\text{object}} + \hat{Q}_{\text{apparatus}}) \geq \frac{1}{2}\left|\langle[\hat{A}, \hat{Q}_{\text{object}} + \hat{Q}_{\text{apparatus}}]\rangle\right|$$

$$= \frac{1}{2}\left|\langle[\hat{A}, \hat{Q}_{\text{object}}]\rangle\right|$$

- Lower bound of the error in the measurement of the observable that does not commute with the additive conserved quantity is given by

$$\varepsilon(\hat{A}) \geq \frac{\left|\langle[\hat{A}, \hat{Q}_{\text{object}}]\rangle\right|^2}{4\left\{\left(\sigma(\hat{Q}_{\text{object}})^2 + \sigma(\hat{Q}_{\text{apparatus}})^2\right)\right\}}$$

Comparison

- The WAY-Ozawa relation holds **when $\hat{\boldsymbol{Q}}_{\mathbf{object}} + \hat{\boldsymbol{Q}}_{\mathbf{apparatus}}$ is conserved** in the process of measurement:

$$\varepsilon(\hat{A}) \geq \frac{\left|\langle[\hat{A},\hat{Q}_{\mathrm{object}}]\rangle\right|^2}{4\left\{\left(\sigma(\hat{Q}_{\mathrm{object}})^2+\sigma(\hat{Q}_{\mathrm{apparatus}})^2\right)\right\}} \neq 0 \quad \Longrightarrow \text{No precise measurements}$$

- The superselection rule holds **when $\hat{\boldsymbol{Q}}_{\mathbf{object}}$ and $\hat{\boldsymbol{Q}}_{\mathbf{apparatus}}$ are conserved individually:**

$$\left[\hat{A},\hat{Q}_{\mathrm{object}}\right] \neq 0 \quad \Longrightarrow \text{ No covariant measurements of } \hat{A}$$

- In this sense, **the superselection rule can be regarded as an extreme case of the uncertainty relation.**

Superselection sectors

- In general, the unitary representation $\hat{S}_g$ of $g \in G$ admits a nontrivial cohomology:

$$\hat{S}_{g_1} \circ \hat{S}_{g_2} = \hat{C}(g_1, g_2)\, \hat{S}_{g_1 \circ g_2}$$

- $\hat{C}(g_1, g_2)$ is commutative with all the measurable observables and with $\{\, \hat{S}_g \mid g \in G\}$.
- (ρ, V_ρ): irreducible projective unitary representation of G. Then

$$\mathfrak{H} = \oplus_\rho\, \mathfrak{H}_\rho = \oplus_\rho\, (V_\rho \otimes W_\rho), \qquad \hat{S}_g = \oplus_\rho\, (\hat{\rho}_g \otimes \hat{1})$$

summation is taken over inequivalent irreducible projective unitary representations.

- Each subspace $\mathfrak{H}_\rho$ defines a sector.

Absence of interference term

- The Hilbert space is decomposed accordingly to irreducible projective unitary representations of G:

$$\mathfrak{H} = \oplus_\rho \mathfrak{H}_\rho = \oplus_\rho \left(V_\rho \otimes W_\rho\right), \qquad \hat{S}_g = \oplus_\rho \left(\hat{\rho}_g \otimes \hat{1}\right)$$

summation is taken over inequivalent irreducible projective unitary representations.

- Under the superselection rule, "measurable observable" $\hat{A}$ satifying $\hat{A}\hat{S}_g = \hat{S}_g\hat{A}$ should have a form $\hat{A} = \oplus_\rho \left(\hat{1} \otimes A_\rho\right)$.
- If ρ_1 and ρ_2 are in-equivalent representation of G, and if $|\psi_1\rangle \in \mathfrak{H}_{\rho_1}$ and $|\psi_2\rangle \in \mathfrak{H}_{\rho_2}$, then

$$\langle \psi_1 + \psi_2 | \hat{A} | \psi_1 + \psi_2 \rangle = \langle \psi_1 | \hat{A} | \psi_1 \rangle + \langle \psi_2 | \hat{A} | \psi_2 \rangle.$$

Interference term $\langle \psi_1 | \hat{A} | \psi_2 \rangle$ between different sectors vanishes.

Summary

- The superselection rule is derived from the viewpoint of measurement.
- When the object system admits action of a group G, if the meter is required to be covariant under the action of G, and if the generator of the G-action is conserved within the object system, the measurable quantity $\hat{A}$ must satisfy the superselection rule, $\hat{S}_g^\dagger \hat{A} \hat{S}_g = \hat{A}$ or $[\hat{Q}_\kappa, \hat{A}] = 0$.
- The superselection rule forbids disturbance of the superselection charge, so it is regarded as an extreme case of the uncertainty relation with additive conservation charge.
- Absence (or non-observability) of interference of different sector is explained.

Remaining problems

- Spontaneous symmetry breaking
- Local gauge symmetry
- Color confinement?

ご清聴ありがとうございました。

Thank you for your attention.